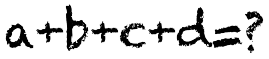

Problem of the Week
Problem C and Solution
Just Sum Primes

Problem

A *prime number* is an integer greater than 1 that has only two positive divisors: 1 and itself. The number 17 is prime because its only positive divisors are 1 and 17.

The variables a, b, c, and d represent four different prime numbers. If $a \times b \times c \times d$ is equal to a three-digit number with a tens digit of 1 and a ones (units) digit of 0, determine all the possible values of $a + b + c + d$.

Solution

Let e be the hundreds digit of the product $a \times b \times c \times d$. In other words, $a \times b \times c \times d = e10$.

Since $e10$ ends in 0, it must be divisible by 10, which is the product of the two primes 2 and 5. That tells us that 2 and 5 must be two of the prime numbers a, b, c, and d.

When $e10$ is divided by 10, the quotient is $e10 \div 10 = e1$. Since $e10$ is a three-digit number, $e \neq 0$ because $e10 = 010 = 10$ is not a three-digit number. Thus, the possibilities for $e1$ are 11, 21, 31, 41, 51, 61, 71, 81, and 91.

The two-digit number $e1$ must be the product of two distinct prime numbers, neither of which is 2 or 5. We can rule out any possibilities for $e1$ that are prime, since these numbers would have only one prime factor. Therefore, we can rule out 11, 31, 41, 61, and 71, which are all prime. The remaining possibilities for e are 2, 5, 8, and 9.

- If $e = 2$, then the two-digit number would be 21, which has prime factors 7 and 3. The four prime factors of $e10 = 210$ are 2, 3, 5, and 7, producing a sum of $2 + 3 + 5 + 7 = 17$.
- If $e = 5$, then the two-digit number would be 51, which has prime factors 17 and 3. The four prime factors of $e10 = 510$ are 2, 3, 5, and 17, producing a sum of $2 + 3 + 5 + 17 = 27$.
- If $e = 8$, then the two-digit number would be 81, but we cannot write 81 as the product of two distinct prime numbers. Note that $810 = 2 \times 3 \times 3 \times 3 \times 3 \times 5$, which is the product of six prime numbers not all of which are distinct. Therefore, 8 is not a possible value for e.
- If $e = 9$, then the two-digit number would be 91, which has prime factors 7 and 13. The four prime factors of $e10 = 910$ are 2, 5, 7, and 13, producing a sum of $2 + 5 + 7 + 13 = 27$. However, we already have the sum 27.

Since there are no other possible cases to consider, the only possible values of $a + b + c + d$ are 17 and 27.