x+y=?

Problem of the Week

Problem D and Solution

Two Equations and Two Variables

Problem

If $2x = 3y + 11$ and $2^x = 2^{4(y+1)}$, determine the value of $x + y$.

Solution

Solution 1

Since $2^x = 2^{4(y+1)}$, it follows that $x = 4(y+1)$, or $x = 4y + 4$. We now have the following two equations.

$$2x = 3y + 11 \qquad (1)$$
$$x = 4y + 4 \qquad (2)$$

We can substitute equation (2) into equation (1) for x.

$$\begin{aligned} 2x &= 3y + 11 \\ 2(4y+4) &= 3y + 11 \\ 8y + 8 &= 3y + 11 \\ 5y &= 3 \\ y &= \frac{3}{5} \end{aligned}$$

Now, we can substitute $y = \frac{3}{5}$ into equation (2) to solve for x.

$$\begin{aligned} x &= 4y + 4 \\ &= 4\left(\frac{3}{5}\right) + 4 \\ &= \frac{12}{5} + \frac{20}{5} \\ &= \frac{32}{5} \end{aligned}$$

Now that we have the values of x and y, we can determine the value of $x + y$.

$$x + y = \frac{32}{5} + \frac{3}{5} = \frac{35}{5} = 7$$

Therefore, the value of $x + y$ is 7.

Solution 2

We can solve this problem in a faster way without finding the values of x and y. Since $2^x = 2^{4(y+1)}$, it follows that $x = 4(y+1)$, or $x = 4y + 4$. We now have the following two equations.

$$2x = 3y + 11 \qquad (1)$$
$$x = 4y + 4 \qquad (2)$$

We can subtract equation (2) from equation (1), and obtain the equation $x = -y + 7$.

Rearranging this equation gives $x + y = 7$. Therefore, the value of $x + y$ is 7.