



## Problem of the Week

### Problem D and Solution

#### Follow the Sequence



#### Problem

The first four terms of an arithmetic sequence are  $x$ ,  $2x$ ,  $y$ , and  $x - y - 6$ , for some integers  $x$  and  $y$ . What is the value of the 50th term in this sequence?

NOTE: An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7, 9 are the first four terms of an arithmetic sequence.

#### Solution

Since each term is obtained by adding the same number to the previous term, then the differences between pairs of consecutive terms are equal. Thus, from the first three terms we can conclude

$$\begin{aligned}2x - x &= y - 2x \\x &= y - 2x \\3x &= y\end{aligned}$$

Now we can substitute  $y = 3x$  into the fourth term to write it in terms of  $x$ .

$$\begin{aligned}x - y - 6 &= x - 3x - 6 \\&= -2x - 6\end{aligned}$$

Therefore, in terms of  $x$ , the first four terms are  $x$ ,  $2x$ ,  $3x$ , and  $-2x - 6$ . However since  $2x - x = x$ , the common difference is  $x$ , so we can also write the fourth term as  $4x$ . Thus,

$$\begin{aligned}4x &= -2x - 6 \\6x &= -6 \\x &= -1\end{aligned}$$

Thus, the first four terms of the sequence are  $-1$ ,  $-2$ ,  $-3$ , and  $-4$ .

To get the 50th term, we must add the common difference 49 times to the first term, to get  $-1 + 49(-1) = -50$ .

Therefore, the 50th term of the sequence is  $-50$ .