Problem of the Week
Problem D and Solution
Sale Boats

Problem

Harold, a marina manager, purchased two boats. He then sold the boats, the first at a profit of 40% and the second at a profit of 60%. The total profit on the sale of the two boats was 54% and \$88 704 was the total selling price of the two boats. What did Harold originally pay for each of the two boats?

Solution

Solution 1

Let a represent what Harold paid for the first boat, in dollars, and b represent what he paid for the second boat, in dollars.

The profit on the sale of the first boat was 40% or $0.4a$ dollars. Thus, the first boat sold for $a + 0.4a = 1.4a$ dollars. The profit on the sale of the second boat was 60% or $0.6b$ dollars. Thus, the second boat sold for $b + 0.6b = 1.6b$ dollars. The total selling price of the two boats was \$88 704, so we have

$$1.4a + 1.6b = 88\,704 \tag{1}$$

Harold bought both boats for a total of $(a + b)$ dollars. The profit on the sale of the two boats was 54% or $0.54(a + b)$ dollars. The two boats sold for $(a + b) + 0.54(a + b) = 1.54(a + b)$ dollars. But the total selling price was \$88 704, so

$$1.54(a + b) = 88\,704$$
$$a + b = 88\,704 \div 1.54$$
$$a + b = 57\,600$$
$$a = 57\,600 - b$$

Substituting $a = 57\,600 - b$ into equation (1) gives

$$1.4(57\,600 - b) + 1.6b = 88\,704$$
$$80\,640 - 1.4b + 1.6b = 88\,704$$
$$0.2b = 8064$$

Dividing by 0.2, we get $b = 40\,320$. Since $b = 40\,320$ and $a + b = 57\,600$, then $a = 17\,280$ follows.

Therefore, Harold paid \$17 280 for the first boat and \$40 320 for the second boat.

Solution 2

Let a represent what Harold paid for the first boat, in dollars, and b represent what he paid for the second boat, in dollars.

The profit on the sale of the first boat was 40% or $0.4a$ dollars. The first boat sold for $a + 0.4a = 1.4a$ dollars. The profit on the sale of the second boat was 60% or $0.6b$ dollars. The second boat sold for $b + 0.6b = 1.6b$ dollars. The total selling price of the two boats was $\$88\,704$ so we have

$$1.4a + 1.6b = 88\,704$$

Multiplying by 5, we get

$$7a + 8b = 443\,520 \tag{1}$$

Harold bought both boats for a total of $(a + b)$ dollars. The profit on the sale of the two boats was 54% or $0.54(a + b)$ dollars. The total profit is the sum of the profit from the sale of each boat, so

$$0.54(a + b) = 0.4a + 0.6b$$
$$0.54a + 0.54b = 0.4a + 0.6b$$
$$0.14a = 0.06b$$

Multiplying by 50, we get

$$7a = 3b \tag{2}$$

Substituting $3b$ for $7a$ into equation (1), we get $3b + 8b = 443\,520$ or $11b = 443\,520$, and $b = 40\,320$ follows.

Substituting $b = 40\,320$ into equation (2), we get $7a = 120\,960$, and $a = 17\,280$ follows.

Therefore, Harold paid $\$17\,280$ for the first boat and $\$40\,320$ for the second boat.