Problem of the Week
Problem D and Solution
Unusually Late

Problem

Every day, a train makes a trip from Alphatown to Betatown. Although the train is rarely late, on two different trips the train was late. On the first trip, when the train was travelling at an average speed of 56 km/h, the train was 27 minutes late. For the second trip, the train was travelling at an average speed of 54 km/h and was 42 minutes late. What is the distance between Alphatown and Betatown?

Solution

We will present three different solutions. In all three solutions, we will use the formula

$$\text{distance} = \text{speed} \times \text{time}$$

or equivalently,

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Solution 1

Let t represent the time, in hours, taken by the train when it was 27 minutes late. Since $42 - 27 = 15$ minutes, then $t + \frac{15}{60} = t + \frac{1}{4}$ represents the time, in hours, taken by the train when it was 42 minutes late, or 15 minutes later.

For the first trip, the speed is 56 km/h and the time is t, and so the distance travelled is $56t$ km.

For the second trip, the speed is 54 km/h and the time is $t + \frac{1}{4}$, and so the distance travelled is $54\left(t + \frac{1}{4}\right)$ km.

Since the distance between Alphatown and Betatown remains constant,

$$\begin{aligned} 56t &= 54\left(t + \frac{1}{4}\right) \\ 56t &= 54t + \frac{27}{2} \\ 2t &= \frac{27}{2} \\ t &= \frac{27}{4} \end{aligned}$$

Thus, the distance between Alphatown and Betatown is $56t = 56 \times \frac{27}{4} = 378$ km.

Solution 2

Let d represent the distance, in km, between Alphatown and Betatown.

For the first trip, the speed is 56 km/h and the distance is d, and so the time for the trip is $\frac{d}{56}$ hours.

For the second trip, the speed is 54 km/h and the distance travelled is d, and so the time for the trip is $\frac{d}{54}$ hours.

Since the difference in times between the first trip and the second trip is $42 - 27 = 15$ minutes or $\frac{1}{4}$ hour,

$$\begin{aligned}
\frac{d}{54} - \frac{d}{56} &= \frac{1}{4} \\
\frac{56d - 54d}{(54)(56)} &= \frac{1}{4} \\
2d &= \frac{1}{4} \times (54)(56) \\
2d &= 756 \\
d &= 378
\end{aligned}$$

Thus, the distance between Alphatown and Betatown is 378 km.

Solution 3

This solution looks at the problem quite differently from the first two solutions.

For the first trip, if the train had first travelled for 27 minutes, it then would have completed the rest of the trip in the usual amount of time. During the 27 minutes, the train would travel $56 \times \frac{27}{60} = \frac{1512}{60} = 25.2$ km.

For the second trip, if the train had first travelled for 42 minutes, it then would have completed the rest of the trip in the usual amount of time. During the 42 minutes, the train would travel $54 \times \frac{42}{60} = \frac{2268}{60} = 37.8$ km.

The slower train is $37.8 - 25.2 = 12.6$ km ahead of the faster train at the point when the usual time to complete the trip remains. The faster train gains 2 km/h on the slower train. Thus, it will take the faster train $\frac{12.6}{2} = 6.3$ h to catch up and thereby complete the trip. In 6.3 h, the faster train travels $56 \times 6.3 = 352.8$ km. But it had already travelled 25.2 km. Therefore, the total distance from Alphatown to Betatown is $25.2 + 352.8 = 378$ km.

Thus, the distance between Alphatown and Betatown is 378 km.