Problem of the Week
Problem C and Solution
Gone Shopping

Problem
While grocery shopping, Terry has a way to approximate the total cost of his purchases. He simply approximates that each item will cost $3.00.

One day, Terry purchased 20 items. He purchased items that each had an actual cost of either $1.00, $3.00, or $7.50. Exactly seven of the purchased items had an actual cost of $3.00. If the total actual cost of the 20 items was the same as the total approximated cost, how many items had an actual cost of $7.50?

Solution
The total approximated cost for the 20 items is $60. Since the total actual cost is the same as the total approximated cost, the total actual cost for the 20 items is $60. Since 7 of the items cost $3.00, it cost Terry $21 to buy these items. Therefore, the remaining 13 items cost $39.

From this point, we will continue with two different solutions.

Solution 1
In this solution, we will use systematic trial-and-error to solve the problem.

Let $s$ represent the number of items Terry bought with an actual cost of $7.50 and $d$ represent the number of items that Terry bought with an actual cost $1.00.

Then the total cost of the $7.50 items would be $7.5s$. Also, the total cost of the $1.00 items would be $1d = d$. Since Terry’s total remaining cost was $39, then $7.5s + d = 39$. We also know that $s + d = 13$.

At this point we can systematically pick values for $s$ and $d$ that add to 13 and substitute into the equation $7.5s + d = 39$ to find the combination that works. (We can observe that $s < 6$ since $7.5 \times 6 = 45 > 39$. If this were the case, then $d$ would have to be a negative number.)

Let’s start with $s = 3$. Then $d = 13 - 3 = 10$. The cost of these items would be $7.5 \times 3 + 10 = 22.50 + 10 = 32.50$, which is less than $39$.

So let’s try $s = 4$. Then $d = 13 - 4 = 9$. The cost of these items would be $7.5 \times 4 + 9 = 30 + 9 = 39$, which is the amount we want.

Therefore, Terry purchased 4 items that cost $7.50.
Solution 2

In this solution, we will use algebra to solve the problem.

Let $s$ represent the number of items that cost $7.50. Therefore, $(13 - s)$ represents the number of items that cost $1.00. Also, the total cost of the $7.50 items would be $7.5s$, the total cost of the $1.00 items would be $1 \times (13 - s) = 13 - s$, and the total of these two is $7.5s + 13 - s = 6.5s + 13$.

Since Terry’s total remaining cost was $39.00, we must have

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6.5s + 13 = 39
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\[
6.5s + 13 - 13 = 39 - 13
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\[
6.5s = 26
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\[
\frac{6.5s}{6.5} = \frac{26}{6.5}
\]
\[
s = 4
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Therefore, Terry purchased 4 items that cost $7.50.