Problem of the Week
Problem D and Solution
Where is Pete?

Problem
Amir, Bita, Colin, and Delilah are standing on the four corners of a rectangular field, with Amir and Colin at opposite corners. Pete is standing inside the field 5 m from Amir, 11 m from Bita, and 10 m from Delilah. In the diagram, the locations of Amir, Bita, Colin, Delilah, and Pete are marked with $A$, $B$, $C$, $D$, and $P$, respectively. Determine the distance from Pete to Colin.

Solution
We start by drawing a line through $P$, perpendicular to $AB$ and $DC$. Let $Q$ be the point of intersection of the perpendicular with $AB$ and $R$ be the point of intersection with $DC$.

Since $QP$ is perpendicular to $AB$, $\angle AQP = 90^\circ$ and $\angle BQP = 90^\circ$. Since $PR$ is perpendicular to $DC$, $\angle DRP = 90^\circ$ and $\angle CRP = 90^\circ$. We also have that $AQ = DR$ and $BQ = CR$.

We can apply the Pythagorean Theorem in $\triangle AQP$ and $\triangle BQP$.

From $\triangle AQP$, we have $AQ^2 + QP^2 = AP^2 = 5^2 = 25$. Rearranging, we have

$$QP^2 = 25 - AQ^2 \quad (1)$$

From $\triangle BQP$, we have $BQ^2 + QP^2 = BP^2 = 11^2 = 121$. Rearranging, we have

$$QP^2 = 121 - BQ^2 \quad (2)$$

Since $QP^2 = QP^2$, from (1) and (2) we find that $25 - AQ^2 = 121 - BQ^2$ or $BQ^2 - AQ^2 = 96$.

Since $AQ = DR$ and $BQ = CR$, this also tells us

$$CR^2 - DR^2 = 96 \quad (3)$$

We can now apply the Pythagorean Theorem in $\triangle DRP$ and $\triangle CRP$. From $\triangle DRP$, we have $DR^2 + RP^2 = DP^2 = 10^2 = 100$. Rearranging, we have

$$RP^2 = 100 - DR^2 \quad (4)$$

When we apply the Pythagorean Theorem to $\triangle CRP$ we have $CR^2 + RP^2 = CP^2$. Rearranging, we have

$$RP^2 = CP^2 - CR^2 \quad (5)$$

Since $RP^2 = RP^2$, from (4) and (5) we find that $100 - DR^2 = CP^2 - CR^2$, or

$$CR^2 - DR^2 = CP^2 - 100 \quad (6)$$

From (3), we have $CR^2 - DR^2 = 96$, so (6) becomes $96 = CP^2 - 100$ or $CP^2 = 196$. Thus $CP = 14$, since $CP > 0$.

Therefore the distance from Pete to Colin is 14 m.