



Problem

Alice has a set of bowls of various sizes. She likes stacking her bowls upside down. A bowl can be *stacked over* another bowl if the smaller bowl can be completely enclosed by the larger bowl. This means the larger bowl can completely hide the smaller bowl.

For the example below, a bowl with a width of 10 cm and height 10 cm can stack over a bowl with a width of 5 cm and a height of 5 cm. In turn they can be stacked over by a bowl with a width of 20 cm and a height of 20 cm. This gives a single stack.



On the other hand, a bowl with a width of 20 cm and a height of 20 cm cannot be stacked over a bowl of a width of 25 cm and a height of 15 cm. Also, a bowl with a width of 25 cm and a height of 15 cm cannot be stacked over a bowl of a width of 20 cm and a height of 20 cm.



Alice has the following set of bowls and starts stacking them. What is the fewest number of stacks that Alice can have?





Solution

We note that the bowl with a width of 45 cm and a height of 45 cm cannot be stacked over a bowl of a width of 50 cm and a height of 30 cm. We also note that we are not able to stack these two bowls the other way either. Therefore, we cannot have a single stack. Thus, if we can find a solution with two stacks then we will find the fewest number of stacks is two.

Here is a solution with two possible stacks. The four bowls in the first row can be stacked and the three bowls in the bottom row can also be stacked.



Therefore, the fewest number of stacks is 2.