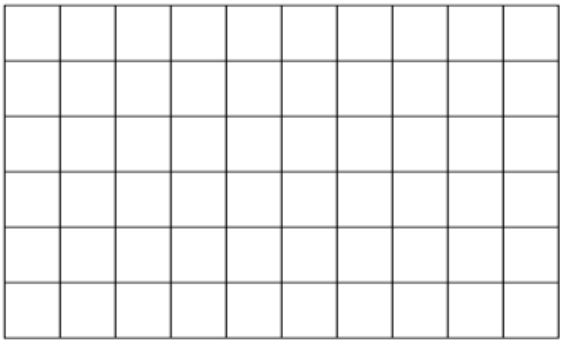
UNIVERSITY OF WATERLOO

Tsunami Detector Placement

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

A 1,000 km by 600 km rectangular area within international waters must be monitored for tsunamis. Suppose a tsunami detector can monitor a circular area within a radius of 200 km.



(a) Determine a way to arrange 12 tsunami detectors so that the entire rectangular area is within range of at least one detector.

(b) What percentage of the rectangular area is within range of exactly one detector?

Connection to the Real World

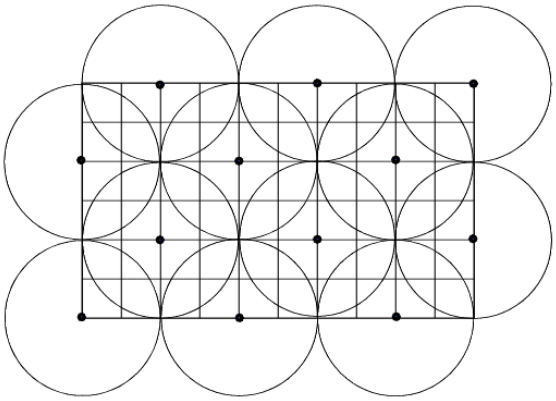
An earthquake under the ocean can trigger a deadly tsunami, washing huge waves of water onto land. Tsunami detectors provide advanced warning of an impending tsunami, saving lives when people are warned to evacuate an area before a tsunami strikes.

Since agencies can afford only a small number of tsunami detectors, it is critical to arrange the detectors to cover as wide an area as possible.

For more Real-World Problems Being Solved by Mathematics, visit http://www.cemc.uwaterloo.ca/resources/real-world.html.

Solution:

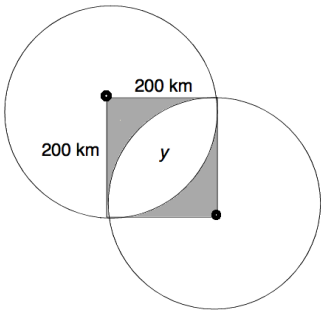
(a) Here is one arrangement, with a detector at the centre of each 200 km radius circle.



(b) The total rectangular area to be monitored measures $1,000$ km $\times 600$ km $= 600,000$ km^2.

To find the area within range of just one detector, we first find the area within range of more than one detector - corresponding to the overlap of circles. Any two circles that overlap have their centres at opposite corners of a square of length 200 km. Label the area of overlap as y; by symmetry, the other two (shaded) areas are equal to each other.

Since the area of each circle is $40,000\pi$ km^2, the area of a quarter circle sector is $10,000\pi$ km^2, and each shaded area is equal to $(10,000\pi - y)$ km^2.
Since the area of the square is 200 km $\times$ 200 km = 40,000 km^2,



$$\begin{aligned} 2(10,000\pi - y) + y &= 40,000 \\ 20,000\pi - 2y + y &= 40,000 \\ y &= 20,000\pi - 40,000 \\ y &= 20,000(\pi - 2) \end{aligned}$$

Within the rectangular area there are 15 such circle overlaps, each having area y km^2.
Therefore the area (in km^2) within range of just one tsunami detector is

$$\begin{aligned} 600,000 - 15y &= 600,000 - 15[20,000(\pi - 2)] \\ &= 120,000 - 300,000\pi \\ &= 300,000(4 - \pi) \end{aligned}$$

The percentage of area represented is thus $\frac{300,000(4-\pi)}{600,000} \times 100\% = \frac{4-\pi}{2} \times 100\% \approx 43\%$.