The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2022 Team Up Challenge

June 2022

Solutions

Team Paper

1. When we arrange the five numbers from least to greatest, we obtain

$$3.0000001, 3.0001, 3.001, 3.01, 3.1$$

Thus, 3.001 is the middle number.

Answer: 3.001

2. A square has exactly four lines of symmetry: one vertical, one horizontal, and two diagonal, as shown in the following diagrams.

We can divide each of these diagrams into nine smaller squares and shade the three in the top row and the three in the bottom row, as shown.

The vertical and horizontal lines still divide the diagram into two halves so that one half is the reflection of the other.
The two diagonal lines divide the diagram into two halves, but the two halves are not reflections of one another any longer.
Thus, the diagram has a vertical line of symmetry and a horizontal line of symmetry, for a total of 2 lines of symmetry.

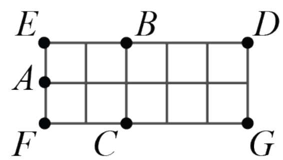
Answer: 2

3. To determine the input, we work backwards through the flowchart.
If the output is 17, then the number before we divide by 2 is 34 since $34 \div 2 = 17$.
Then the number before we subtract 6 is 40 since $40 - 6 = 34$.
Finally, the number before we multiply by 5, which is the input, is 8 since $8 \times 5 = 40$.

Answer: 8

4. We can show that the triangle with vertices B, C, and D has an area of 3 square units. The base BD has length 3 and the height BC is 2, and so the area of $\triangle BCD$ is $\frac{1}{2} \times 3 \times 2 = 3$ square units.

   For completeness, we show that the other triangles do not have an area of 3 square units.
   In $\triangle ABC$, the base BC has length 2 and the perpendicular height of the triangle from BC to A is also 2. Thus, the area of $\triangle ABC$ is $\frac{1}{2} \times 2 \times 2 = 2$ square units.
   In $\triangle ABD$, the base BD has length 3 and the perpendicular height of the triangle from BD to A is 1. Thus, the area of $\triangle ABD$ is $\frac{1}{2} \times 3 \times 1 = 1.5$ square units.
   To find the area of $\triangle ACD$, let the top left, bottom left, and bottom right corners of the grid be E, F, and G, respectively.



   Then the area of $\triangle ACD$ is equal to the area of rectangle $EFGD$ minus the areas of $\triangle AED$, $\triangle DGC$, and $\triangle AFC$.
   The area of rectangle $EFGD$ is $5 \times 2 = 10$ square units.
   The area of $\triangle AED$ is $\frac{1}{2} \times 5 \times 1 = 2.5$ square units.
   The area of $\triangle DGC$ is $\frac{1}{2} \times 3 \times 2 = 3$ square units.
   The area of $\triangle AFC$ is $\frac{1}{2} \times 2 \times 1 = 1$ square units.
   Thus, the area of $\triangle ACD$ is $10 - 2.5 - 3 - 1 = 3.5$ square units.

   Answer: B, C, D

5. Reading from the bar graph, there were 12 hats sold that were size L.
   Since half the hats sold were size L, the total number of hats sold was $2 \times 12 = 24$ hats.
   From the graph, we can see that 6 hats were sold in size S and 4 hats sold in size M.
   Thus, a total of $6 + 4 + 12 = 22$ hats sold were either S, M, or L.
   This means that $24 - 22 = 2$ hats were sold in size XL.

   Answer: 2

6. The three flowering plants can end up next to each other by

   - first swapping the leftmost flowering plant once towards the right, and then
   - swapping the rightmost flowering plant three times towards the left.

   Every non-flowering plant must be involved in a swap because each non-flowering plant is positioned between two flowering plants. Swapping two non-flowering plants does not have any effect so each useful swap of a non-flowering plant must be with a flowering plant.
   There are four non-flowering plants and so there must be at least four swaps.
   Notice that moving the flowering plants all the way left or all the way right requires more than four swaps.

   Answer: 4

7. The even integers between 1 and 53 are $2, 4, 6, \ldots, 52$. Since there are 26 integers in the list $2, 4, 6, \ldots, 50, 52$, then there are 26 even integers between 1 and 53.
   Next, we want to find a number n such that there are 26 odd integers between 12 and n.
   We notice that our lower bound, 12, is 11 greater than our original lower bound of 1.
   By increasing each of the 26 even integers from above by 11, we create the first 26 odd integers which are greater than 12.
   These odd integers are $2 + 11 = 13$, $4 + 11 = 15$, $6 + 11 = 17$, and so on up to and including $52 + 11 = 63$. Since there are 26 odd integers in the list $13, 15, 17, \ldots, 63$, then there are 26 odd integers between 12 and 64.
   That is, the number of even integers between 1 and 53 is the same as the number of odd integers between 12 and 64.

   Answer: 64

8. First, Chance collects 11 acorns per trip for as long as he can. We use division to calculate how many times he can do this:

$$31 = 2 \times 11 + 9$$
$$3 = 0 \times 11 + 3$$
$$35 = 3 \times 11 + 2$$
$$28 = 2 \times 11 + 6$$
$$26 = 2 \times 11 + 4$$

   After $2 + 0 + 3 + 2 + 2 = 9$ trips, Chance has collected $9 \times 11 = 99$ acorns.
   On his 10^{th} trip, Chance collects the largest number of leftover acorns, which is 9 acorns.
   In total, Chance collects $9 \times 11 + 9 = 108$ acorns.

   Answer: 108

9. The first row is missing a 1 and a 3. Since there is already a 3 in the second column (in the second row), the first row, second column must contain a 1 and the first row, fourth column must contain a 3.
   To complete the upper left 2×2 square, the second row, first column must contain a 4.
   The second row is now missing a 1 and a 2. But the fourth column already contains a 1 (in the fourth row), therefore the second row, fourth column must contain a 2.
   To complete the fourth column, we place a 4 in the third row.
   Now the D cannot be a 4, since there is already a 4 in the third row. Also, the D cannot be a 1 or a 3, since the second column already contains these numbers. By process of elimination, the digit 2 must replace the D.
   The completed grid is shown.

2	1	4	3
4	3	1	2
1	2	3	4
3	4	2	1

   Answer: 2

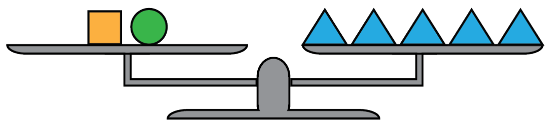
10. The second scale shows that ■ has the same mass as ▲▲▲.

    If we double what is on both sides of this scale, then ■■ has the same mass as ▲▲▲▲▲▲.

    From the first scale, ■■ also has the same mass as ●●●.

    Therefore, ●●● must have the same mass as ▲▲▲▲▲▲, or ● must have the same mass as ▲▲.

    Since ● has the same mass as ▲▲ and ■ has the same mass as ▲▲▲ it follows that ■● has the same mass as ▲▲▲▲▲, as shown.



    The third scale shows that ■● also has the same mass as ★.

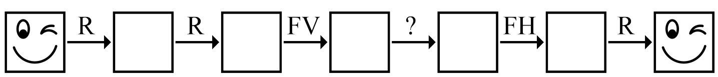
    So ★ must have the same mass as ▲▲▲▲▲.

    If ★ has a mass of 10 grams, then ▲▲▲▲▲ must have a mass of 10 grams.

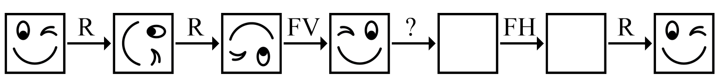
    Since each triangle has the same mass, ▲ has a mass of 2 grams.

Answer: 2

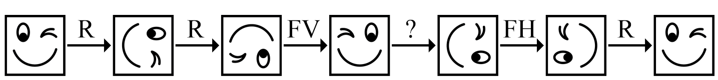
11. In total, six actions are performed. Five of the actions are described and one is unknown as a result of the missing block of code. After each action, the face's orientation changes.
    In the following diagram, seven squares are arranged from left to right, which will represent the seven orientations the face takes as the code is executed. Since the face starts and ends in the same orientation, we draw the same face in the first and last squares. In between two side-by-side squares is an arrow pointing right. Over top of the arrows we write the block names (using FV, FH, and R for short) in the order that the code executes them. The question mark over top of the fourth arrow indicates the missing block of code.



    Following the code from the beginning, we can determine the orientation of the face after the first three actions have been performed.



    After the missing block, two more actions are performed and the final orientation of the face is the same that it started in. Working backwards, we can determine the orientation of the face in the remaining two positions.



    Examining the face in the fourth and fifth position, we can conclude that the missing block should be rotate.

Answer: rotate

12. The volume of the cheese before Raissa's cut is $6 \times 7 \times 8 = 336$ cm^3.
The volume of the remaining cheese is 309 cm^3, so the cube has a volume $336 - 309 = 27$ cm^3.
Since $3 \times 3 \times 3 = 27$, a cube with a volume of 27 cm^3 has a side length of 3 cm.

Answer: 3

13. Since the digit 8 appears twice, there are six different configurations that the two 8 digits can be in.

8 8 __ __	__ 8 8 __	8 __ 8 __
__ __ 8 8	8 __ __ 8	__ 8 __ 8

For each of these six configurations, there are two ways for the digits 1 and 4 to be placed (either with 1 first or with 4 first).
Therefore, there are $6 \times 2 = 12$ possible codes.
At the very most, Anil will have to try all 12 codes before he opens the lock. Since it takes Anil 5 seconds to try one code, it takes him $5 \times 12 = 60$ seconds to try all 12 codes.

Answer: 60

14. If Mirela and Kumara have sold 90% of a total of 390 bracelets, then they have sold $0.9 \times 390 = 351$ bracelets.
Since 207 of the bracelets sold were made by Kumara, then $351 - 207 = 144$ of the bracelets sold were made by Mirela.
But, 80% of bracelets made by Mirela were sold, and so 80% of Mirela's bracelets equals 144.
Since 80% of 180 equals 144, Mirela must have made a total of 180 bracelets.

Answer: 180

15. Since there are eight given integers, the smallest nada set could contain one integer and the largest nada set could contain eight integers.

Case 1: Sets containing one integer
All of the given integers are non-zero so Elise cannot make any nada sets containing exactly one integer.

Case 2: Sets containing two integers
The given integers do not contain any pairs that add to zero, so Elise cannot make any nada sets containing two integers.

Case 3: Sets containing three integers

If three integers have a sum of zero then either one integer is positive or two integers are positive. Can you verify this for yourself?

- What if one integer is positive?
  If one integer is positive then the other two must be negative. The possible sums of the two negative integers are:

$$(-1) + (-8) = (-9) \qquad (-1) + (-11) = (-12) \qquad (-1) + (-3) = (-4)$$
$$(-8) + (-11) = (-19) \qquad (-8) + (-3) = (-11) \qquad (-11) + (-3) = (-14)$$

  Notice that in no way can one of the positive integers $2, 5, 6$, or 10 be added to any of these sums to yield a total sum of zero.
  There are no nada sets containing three integers where one is positive.

- What if two integers are positive?
  If two integers are positive then the third integer must be negative. The possible sums of the two positive integers are:

$$5 + 6 = 11 \qquad 5 + 2 = 7 \qquad 5 + 10 = 15$$
$$6 + 2 = 8 \qquad 6 + 10 = 16 \qquad 2 + 10 = 12$$

  Notice there are two different ways that one of the negative integers $-1, -3, -8$, or -11 can be added to one of these sums to yield a total sum of zero.
  Thus, $5, 6, -11$ and $2, 6, -8$ are the two sets of three integers that have a sum of zero.

Elise can make two different nada sets containing three integers.

Case 4: Sets containing four integers

If a set of four integers has a sum of zero then either one integer is positive, two integers are positive, or three integers are positive.

- What if one integer is positive?
  If one integer is positive then the other three integers must be negative.
  The positive integers are between 2 and 10.
  The possible sum of three negative integers is between $(-3) + (-8) + (-11) = (-22)$ and $(-1) + (-3) + (-8) = (-12)$.
  Thus, any sum of one positive integer and three negative integers will always be less than zero, so no such nada sets can be found.

- What if two integers are positive?
  If a set of two positive and two negative integers has a sum of zero, then the sum of the positive integers must be opposite to the sum of the negative integers.
  Using the sums found in Case 3, we have that $5, 6, -3, -8$ and $2, 10, -1, -11$ are the two sets of four integers that have a sum of zero.

- What if three integers are positive?
  If three integers are positive then the fourth integer must be negative.
  The possible sum of three positive integers is between $2 + 5 + 6 = 13$ and $5 + 6 + 10 = 21$.
  The negative integers are between -11 and -1.
  Thus, any sum of three positive integers and one negative integer will always be greater than zero, so no such nada sets can be found.

Elise can make two different nada sets containing four integers.

Case 5: Sets containing five integers
The given eight integers have a total sum of zero. Thus, if a set of five integers has a sum of zero, then the remaining three integers must also have a sum of zero.
Can you verify this for yourself?
From before, we know that there are two different sets of three integers that have a sum of zero, namely $5, 6, -11$ and $2, 6, -8$.
Thus, there are also two different sets of five integers that have a sum of zero, namely $-1, -8, 2, -3, 10$ and $-1, 5, -11, -3, 10$.
Elise can make two different nada sets containing five integers.

Case 6: Sets containing six integers
Similarly, since the sum of a set containing any two of the given integers cannot equal zero, it follows that the sum of a set containing any six of the given integers cannot equal zero.
Elise cannot make any nada sets containing six integers.

Case 7: Sets containing seven integers
Since the sum of a set containing any one of the given integers cannot equal zero, it follows that the sum of a set containing any seven of the given integers cannot equal zero.
Elise cannot make any nada sets containing seven integers.

Case 8: Sets containing eight integers
The sum of all eight integers equals zero, so Elise can make one nada set containing eight integers.

Therefore, Elise can make $0 + 0 + 2 + 2 + 2 + 0 + 0 + 1 = 7$ different nada sets using the integers on the cards.

ANSWER: 7

Crossnumber Puzzle

5		2	7	7		1	0	8
8	5	7		9	7	8		2
5			3		5		3	6
	1	9	9	1		9	9	0
4	5		5		3		2	4
5	8	7		5	3	4	6	
6	0		2		6			1
7		1	9	6		6	1	6
8	8	1		4	6	6		2

Across

2. From the grid, the hundreds digit of this number is 2 and the ones digit is 7. Since the mode of the digits is 7, the tens digit must be 7. Thus, this number is 277.

4. From the grid, the hundreds digit of this number is 1. The only number that is a multiple of both 27 and $\boxed{36}$, that also has a hundreds digit of 1, is 108.

6. The positive difference is $\boxed{881} - \boxed{24} = 857$.

7. The median of any three digits is an integer, and so the mean must also be an integer. From the grid, the hundreds digit of this number is 9 and the ones digit is 8. For the mean to be an integer, the tens digit must be $1, 4$, or 7.
If the tens digit is 1 or 4 then the median and the mean of the three digits are not equal.
The tens digit must be 7. Thus, the number is 978.

10. The last two digits of $\boxed{336}$ are 36.

11. From the grid, the thousands digit of this number is 1. It follows that the ones digit is 1. From the grid, the tens digit is 9. It follows that the hundreds digit is 9.
Thus, the number must be 1991.

12. From the grid, the tens digit of this number is 9. The only three-digit number with a tens digit of 9 that is the product of three consecutive integers is $9 \times 10 \times 11 = 990$.

13. From the grid, the ones digit of this number is 5. The only two-digit number with a ones digit of 5 that is a multiple of 9 is 45.

15. One square has 4 sides, so six squares have a total of $6 \times 4 = 24$ sides.

16. The number of nickels is $\$29.35 \div \$0.05 = 587$.

17. The volume is equal to $\boxed{18} \times \boxed{11} \times \boxed{27} = 5346$.

18. Since $120 \div 60 = 2$ and $180 \div 60 = 3$, the largest number that both 120 and 180 are divisible by is 60.

21. From the grid, the hundreds digit of this number is 1 and the tens digit is 9. Since $49 \times 4 = 196$, this number is 196.

23. $6 \times 100 + 1 \times 10 + 6 = 600 + 10 + 6 = 616$.

24. The sum of the digits in $\boxed{395}$ is $3 + 9 + 5 = 17$. From the grid, the hundreds digit of this number is 8 and the ones digit is 1. Since $17 - 8 - 1 = 8$, this number is 881.

25. From the grid, the hundreds digit of this number is 4 and the ones digit is 6. The only three-digit number with a hundreds digit of 4 and a ones digit of 6 that is equal to $\boxed{587}$ minus an integer multiplied by itself is $\boxed{587} - 11 \times 11 = \boxed{587} - 121 = 466$.

Down

1. From the grid, the tens digit of this number is 8. Since $200 \div 8 = 25$, and the only two digits that multiply to 25 are 5 and 5, this number is 585.

2. From the grid, the ones digit of this number is 7. The only possible tens digit that produces a number equal to three times the sum of its digits is 2, since $3 \times (2 + 7) = 3 \times 9 = 27$. So this number is 27.

3. 5% of $\boxed{1580} = 0.05 \times \boxed{1580} = 79$.

4. Two-fifths of $\boxed{45} = \frac{2}{5} \times \boxed{45} = 18$.

5. From the grid, the digits 8, 6, 0, and 4 have been used so far. The only remaining even digit is 2. So this number is 82 604.

8. From the grid, the tens digit of this number is 7. The only odd multiple of 3 with a tens digit of 7 is 75.

9. The product is $5 \times 79 = 395$.

10. $2 \times \boxed{1991} - 56 = 3982 - 56 = 3926$.

11. In 1 centimetre there are 10 millimetres, and so it follows that in 158 centimetres there are $10 \times 158 = 1580$ millimetres.

13. From the grid, the thousands digit of this number is 5. Since each digit is one more than the digit before it, this number is 45 678.

14. In 1 day there are 24 hours, and so it follows that in 14 days there are $24 \times 14 = 336$ hours.

19. The smallest prime number greater than $\boxed{24}$ is 29.

20. Since $\frac{1}{9} = \frac{\boxed{18}}{162}$, the number is 162.

21. The only factors of $\boxed{1991}$ are $1,\ 11,\ 181$, and 1991. The only two-digit factor is 11.

22. From the grid, the tens digit of this number is 6. The only two-digit number with a tens digit of 6 that is the product of two equal integers is 64.

23. A triangle with area $\boxed{5346}$ and base $\boxed{162}$ has height 66 since $\frac{1}{2} \times \boxed{162} \times 66 = \boxed{5346}$. Thus, the number is 66.

Logic Puzzle

We start by considering clues (2) and (5):

(2) *Damien feeds the sheep on only Monday and Wednesday.*

(5) *Damien feeds only the pigs and the sheep.*

Since Damien feeds the sheep on only Monday and Wednesday, and Damien feeds only the pigs and the sheep, it follows that Damien must feed the pigs on Tuesday, Thursday, and Friday.

Next we consider clue (6):

(6) *Every day that Damien feeds the pigs, Corina feeds the goats.*

Since Damien feeds the pigs on Tuesday, Thursday, and Friday, it follows that Corina feeds the goats on Tuesday, Thursday, and Friday.

Next we consider clue (8):

(8) *The student who feeds the cows on Monday also feeds the goats on Tuesday.*

Since Corina feeds the goats on Tuesday, it follows that Corina must feed the cows on Monday.

The following partially-completed table contains the information we have determined so far.

		Day of the Week				
		Monday	Tuesday	Wednesday	Thursday	Friday
Kind of Animal	cows	**Corina**				
	goats		**Corina**		**Corina**	**Corina**
	pigs		**Damien**		**Damien**	**Damien**
	sheep	**Damien**		**Damien**		
	horses					

Next we consider clues (3), (1), (4), and (10):

(3) *The same student feeds the horses on Monday, Tuesday, and Wednesday.*

(1) *The student who feeds the cows on Thursday also feeds the horses on Wednesday.*

(4) *Eliel feeds all the animals except for the cows.*

(10) *Aditi does not feed the same kind of animal more than twice each week.*

Clues (3) and (1) tell us that one particular student feeds the horses on Monday, Tuesday, and Wednesday, and the cows on Thursday. Clues (4) and (10) tell us that this student is not Eliel or Aditi. On Monday Corina and Damien are already feeding other animals, so the only student left is Brigid. Thus, Brigid must feed the horses on Monday, Tuesday, and Wednesday, and the cows on Thursday.

Now on Tuesday, the only remaining students are Aditi and Eliel, and the only remaining animals are the cows and the sheep. From clue (4) we can determine that on Tuesday, Aditi feeds the cows and Eliel feeds the sheep.

Next, we consider clue (9):

(9) *The student who feeds the goats on Monday also feeds the cows on Tuesday.*

Since Aditi feeds the cows on Tuesday, it follows that Aditi feeds the goats on Monday. Then Eliel is the only student left on Monday, so Eliel must feed the pigs on Monday.

The following partially-completed table contains the information we have determined so far.

		Day of the Week				
		Monday	Tuesday	Wednesday	Thursday	Friday
Kind of Animal	cows	**Corina**	**Aditi**		**Brigid**	
	goats	**Aditi**	**Corina**		**Corina**	**Corina**
	pigs	**Eliel**	**Damien**		**Damien**	**Damien**
	sheep	**Damien**	**Eliel**	**Damien**		
	horses	**Brigid**	**Brigid**	**Brigid**		

Next we look back at clue (4):

(4) *Eliel feeds all the animals except for the cows.*

This clue tells us that Eliel feeds the goats at least one day of the week. Since Wednesday is the only day left for the goats to be fed, it follows that Eliel must feed the goats on Wednesday.

Next we look at clue (7):

(7) *The sheep are fed by four different students each week.*

The sheep are being fed by Damien and Eliel on Monday, Tuesday, and Wednesday. Thus, on Thursday and Friday the sheep must be fed by two different students, who are not Damien or Eliel. On Thursday, the only available students are Aditi and Eliel, and so Aditi must feed the sheep leaving Eliel to feed the horses. On Friday, the only available students are Aditi, Brigid, and Eliel. The only one of those who has not already been assigned to the sheep is Brigid, so Brigid must feed the sheep on Friday.

From clue (4) we know that Eliel does not feed the cows, so on Friday, Eliel must feed the horses and Aditi must feed the cows.

Finally, we look at clue (10):

(10) *Aditi does not feed the same kind of animal more than twice each week.*

On Wednesday, the only available students are Aditi and Corina, and the only available animals are the pigs and the cows. Since Aditi feeds the cows on Tuesday and Friday, it follows that he cannot also feed them on Wednesday. So on Wednesday Aditi must feed the pigs and Corina must feed the cows.

This completes the logic puzzle.

		Day of the Week				
		Monday	Tuesday	Wednesday	Thursday	Friday
Kind of Animal	cows	**Corina**	**Aditi**	**Corina**	**Brigid**	**Aditi**
	goats	**Aditi**	**Corina**	**Eliel**	**Corina**	**Corina**
	pigs	**Eliel**	**Damien**	**Aditi**	**Damien**	**Damien**
	sheep	**Damien**	**Eliel**	**Damien**	**Aditi**	**Brigid**
	horses	**Brigid**	**Brigid**	**Brigid**	**Eliel**	**Eliel**

Relay

(Note: Where possible, the solutions are written as if the value of N is not initially known, and then N is substituted at the end.)

Practice Relay

P1: Evaluating, $1+2+3+4+5=15$.

P2: The fruits that are *not* apples are the oranges, pears, and bananas. The total number of these is $6+N+4=10+N$.
Since the answer to the previous question is 15, then $N=15$, and so $10+N=10+15=25$.

P3: The numbers 15, 40, 105, and 140 are the only numbers in the list that are divisible by 5.
So if N is not divisible by 5 then there are 4 numbers in the list that are divisible by 5.
If N is divisible by 5 then there are 5 numbers in the list that are divisible by 5.
Since the answer to the previous question is 25, then $N=25$. Since 25 is divisible by 5, it follows that there are 5 numbers in the list that are divisible by 5.

P4: After the elevator went up 2 floors it was on floor $N+2$. After it went down 1 floor it was on floor $N+2-1=N+1$. After it went down 3 more floors it was on floor $N+1-3=N-2$.
Since the answer to the previous question is 5, then $N=5$, and so $N-2=5-2=3$.

ANSWER: 15, 25, 5, 3

Relay A

P1: The number line shown has length $10-1=9$.
The number line is divided into three equal parts, and so each part has length $9\div 3=3$.
x is positioned one part after 1, and so the value of x is $1+3=4$.

P2: After Josh presses the button once, there will be $N\times 2$ jellybeans.
After he presses the button twice, there will be $N\times 2\times 2=N\times 4$ jellybeans.
After he presses the button three times, there will be $N\times 4\times 2=N\times 8$ jellybeans.
Since the answer to the previous question is 4, then $N=4$, and so $N\times 8=4\times 8=32$.

P3: There are 5 numbers in the list of numbers that is repeated.
The sum of the numbers in the list is $1+39+24+16+N=80+N$.
The first 15 numbers in the sequence will contain this list of numbers three times in a row.
The sum of the first 15 numbers is $3\times(80+N)$.
The 16th number in the sequence is 1. So the sum of the first 16 numbers is $3\times(80+N)+1$.
Since the answer to the previous question is 32, then $N=32$, and so the sum of the first 16 numbers is $3\times(80+N)+1=3\times(80+32)+1=3\times 112+1=337$.

P4: The smallest positive integer greater than 99 that doesn't have any repeated digits is 102.
The largest two-digit positive integer that is divisible by 4 is 96.
The sum of these three values is $102+96+(3\times N)=198+(3\times N)$.
Since the answer to the previous question is 337, then $N=337$, and so the sum is $198+(3\times 337)=198+1011=1209$.

ANSWER: 4, 32, 337, 1209

Relay B

P1: Since $\triangle \times \triangle = 9$, it follows that $\triangle = 3$.
Then, $\triangle + 5 = 3 + 5 = 8$, so $\bigcirc = 8$.
Then, $\bigcirc \times \triangle = 8 \times 3 = 24$. So $\square = 24$.

P2: In total, Stefanie gave the group $(2 \times 8) + (3 \times 10) + (3 \times 12) = 16 + 30 + 36 = 82$ cookies.
Since the answer to the previous question is 24, then $N = 24$.
Since $24 \times 3 = 72 < 82$, and $24 \times 4 = 96 > 82$, it follows that the maximum number of cookies that each person could have taken is 3.

P3: The width of the rectangle is $9 - 5 = 4$ units. The length of the rectangle is $10 - N$ units. The area of the rectangle is then $4 \times (10 - N)$ units^2.
Since the answer to the previous question is 3, then $N = 3$, and so the area of the rectangle is $4 \times (10 - N) = 4 \times (10 - 3) = 4 \times 7 = 28$ units^2.

P4: Since there are 3 black marbles, then there are $3 \times 2 = 6$ green marbles. Then there are $6 \times 2 = 12$ yellow marbles. The total number of black, green, and yellow marbles is then $3 + 6 + 12 = 21$. The number of red marbles is therefore equal to $N - 21$.
Since the answer to the previous question is 28, then $N = 28$, and so $N - 21 = 28 - 21 = 7$.

Answer: 24, 3, 28, 7

Relay C

P1: If we choose the coins with the largest possible values first, then we will use the fewest number of coins. This can be done using two quarters, one dime, and one nickel.
The total value of these coins is $(2 \times 25) + (1 \times 10) + (1 \times 5) = 50 + 10 + 5 = 65$ cents.
There are 4 coins in total.

P2: The distance between the houses is $2 + 3 + 1.5 + N + 1.5 = 8 + N$ kilometres.
If Vivek bikes there and back, then in total, Vivek bikes $2 \times (8 + N)$ kilometres.
Since the answer to the previous question is 4, then $N = 4$, and so Vivek bikes a total of $2 \times (8 + N) = 2 \times (8 + 4) = 2 \times 12 = 24$ kilometres.

P3: Since Anar plants 12 trees, then Bette plants $\frac{2}{3} \times 12 = 8$ trees. Then Cleo plants $3 + 8 = 11$ trees and in total they plant $12 + 8 + 11 = 31$ trees every day.
After N days they have planted $31 \times N$ trees. Since the answer to the previous question is 24, then $N = 24$, and so $31 \times N = 31 \times 24 = 744$ trees.

P4: We can convert each of the units so that each mass is written in grams.

$$840 \text{ g}, \ 135 \text{ g}, \ 1.945 \text{ g}, \ 1765 \text{ g}, \ 1500 \text{ g}, \ N \text{ g}$$

If $N < 1765$, then the mass of the heaviest object is 1765 g.

If $N > 1765$, then the mass of the heaviest object is N g.

Since the answer to the previous question is 744, then $N = 744$, and since $744 < 1765$, the mass of the heaviest object is 1765 g.

Answer: 4, 24, 744, 1765